Section 11.3: Geometric Sequences and Series

Video 1: Geometric Sequences

A **geometric sequence** is a sequence in which each term is equal to the preceding term multiplied by a fixed nonzero constant (**common ratio, *r***).

$$r = \frac{a_{n+1}}{a_n}$$

General (*n*th) Term of a Geometric Sequence

$$a_n = a_1 \cdot r^{n-1}$$

The common difference of an arithmetic sequence is given by $d = a_n - a_{n-1}$.

1) A person receives a gift on January 1 of $10. On the first day of each subsequent month, the amount of the person's gift doubles.

Find the general term for this geometric sequence, and determine how much the person will receive on September 1.

2) Find a_6 and a_n for the geometric sequence 3, 15, 75, 375, …

3) Find the first 5 terms of the geometric sequence with $a_1 = 4$ and $r = \frac{1}{2}$.

Find the formula for the general term of this sequence.

4) Find a_1 and *r* for the geometric sequence with $a_4 = 40$ and $a_7 = 320$.

Video 2: Geometric Series

If a geometric sequence has first term a_1 and common ratio *r*, the sum S_n of the first *n* terms is given by:

$$S_n = \frac{a_1 \cdot \left(1 - r^n\right)}{1 - r} \qquad \text{where } r \neq 1$$

5) A person receives a gift on January 1 of $10. On the first day of each subsequent month, the amount of the person's gift doubles.

Find the general term for this geometric sequence, and determine how much the person will receive from January 1 through September 1.

6) Find S_8 for the geometric sequence 6, 18, 54, …

7) Evaluate $\sum_{i=1}^{5} 3 \cdot 6^i$

Video 3: Infinite Geometric Series

If $|r| < 1$, the sum of an infinite geometric series, S_∞, is given by the formula $S_\infty = \dfrac{a_1}{1-r}$.

If $|r| \ge 1$, then the sum of the infinite series cannot be found.

8) Find S_∞ for a geometric sequence with $a_1 = 8$ and $r = \dfrac{1}{4}$.

9) Evaluate each sum.

a) $\sum_{i=1}^{\infty} 0.5^i$

b) $\sum_{i=1}^{\infty} \frac{2}{3} \cdot \left(-\frac{1}{6}\right)^{i-1}$

Video 4: Future Value of an Annuity

10) A person deposits $1200 in an account at the end of each year for 8 years.

If the account pays 6% annual interest, compounded annually, find the future value of this annuity at the end of 8 years.